ARTICLE NO. 78-1205

Application of the Vortex-Lattice Technique to Arbitrary Bodies

K. R. Asfar*, D. T. Mook†, and A. H. Nayfeh‡
Virginia Polytechnic Institute and State University, Blacksburg, Va.

A numerical technique that predicts the potential flowfield past arbitrary bodies is presented. To represent the body surface, the technique utilizes a combination of a vortex lattice and sources. The vortex-lattice is formed by constant-strength quadrilateral, as well as triangular, elements. The technique features the minimum self-induced-velocity control point, which was found to be the best choice, and uses the element loop circulations as the unknowns while working with predetermined surface-source strengths. For blunt bodies, the combination of a vortex lattice and sources appears to be superior to either the votex lattice or the sources acting alone. For slender bodies, the combination offers no apparent advantage over the vortex lattice alone, and the vortex lattice appears to be superior to the sources. This technique was tested by comparing the predicted pressures with exact solutions and experimental data. The present method can reliably and accurately treat flows past arbitrary bodies.

I. Introduction

WE consider potential flows past three-dimensional arbitrary bodies. Hess and Smith[1] developed a computer program that uses a surface-source distribution to solve this problem. They used constant-strength panels. Later, Hess and Martin[2] modified this approach by using parabolic elements and linearly-varying source densities. They obtained a considerable reduction in time for a given accuracy. Maskew[3] and Atta and Nayfeh[4] used a surface distribution of sources, sinks and vortices to simulate arbitrary flows past bodies of revolution. He used weighting factors to combine the singularities. According to Geissler, these weighting factors are complicated functions of the body geometry. Bristow[6] used a surface combination of sources and vortices for the analysis of lifting airfoils for two-dimensional potential flow. He used a mean square singularity density minimizatino scheme. Uchiyama et al.[7] treated a wing-body combination by representing the wing by a vortex-lattice and the body by a source distribution. Nikolitsch[8] analyzed a wing-body combination at high angles of attack (up to 40 deg) by treating the body according to Wardlaw's multivortex model and the wing according to Gerstein's nonlinear lifting surface theory. Morino[9] used a finite-element formulation, and David and Geppson[10] used a finite-difference formulation in other attempts to treat this problem.

In the present technique, we use a combination of surface-sources and a vortex-lattice to simulate potential flows arbitrary three-dimensional bodies. The strength of the surface-source distribution is prescribed in such a way that the sources on any given element, when acting alone, generate a velocity field which cancels a prescribed fraction of the normal component of the freestream velocity at the control point of that element. By specifying the source strength, we do not introduce any new unknowns (i.e., we solve for the vorticity distribution in the same manner as we do when the sources are absent). Moreover, many of the computations needed to obtain the velocity field generated by the source distribution are also needed to obtain the velocity field generated by the vortex-lattice. Thus, the addition of sources requires practically no additional effort.

II. Description of the Method

Mathematical Statement of the Problem

The steady potential flow past general bodies is governed by Laplace's equation

$$\nabla^2\phi=0 \tag{1}$$

and the following boundary conditions: the no-penetration condition on the surface of the body

$$\nabla\phi\cdot\boldsymbol{n}=-\boldsymbol{U}_\infty\cdot\boldsymbol{n} \tag{2}$$

and the vanishing of the disturbance far from the body

$$\nabla\phi\rightarrow 0 \tag{3}$$

where

ϕ = the disturbance velocity potential function
n = the unit normal on the surface of the body, and
U_∞ = the freestream velocity

Basic Concepts

The present method is based upon what is derived from Green's theorem. The theorem tells us that the velocity potential at any point in the flowfield can be expressed as the effects of source and doublet (or vortex) sheets distributed on the surface of the body under consideration. The velocity fields generated by the sources and vorticity satisfy Eqs. (1 and 3), regardless of their strengths. It only remains, therefore, to choose their strengths so that Eq. (2) is satisfied. Both a source distribution alone and a vorticity distribution alone can be made to satisfy Eq. (2), and one is free to choose one and solve for the other. Here we elect to specify the source strength and solve for the vortex strength using Eq. (2).

Numerical Procedures

The body under consideration is described by a number of points on its surface. These points could be fed into the computer in the form of an input-data set, or they could be generated by the computer if the geometry of the body surface

---

Presented as Paper 78-1205 at the AIAA 11th Fluid and Plasma Dynamics Conference, Seattle, Wash., July 10-12, 1978; submitted Sept. 11, 1978; revision received Jan. 12, 1979.

Index categories: Aerodynamics; Hydrodynamics.

*Graduate Research Assistant, Engineering Science and Mechanics Dept.

†Professor, Engineering Science and Mechanics Dept. Member AIAA.

‡University Distinguished Professor, Engineering Science and Mechanics Dept. Member AIAA.

is described by known analytic functions. The points on the surface are organized in "rows" and "columns." The lattice consists of short straight lines connecting these points, thereby forming quadrilateral elements with the exception of certain regions where these lines converge and the elements are triangular. Such a point is referred to as a "pole." Along each side of every element is a vortex filament having constant circulation. Within each element is a constant-strength surface source. The numbers of rows and columns are arbitrary, but the technique works better when they are chosen such that the elements are more or less equilateral (See Fig. 1).

For certain symmetric bodies, with either one plane or two planes of symmetry, the number of columns must be an odd number such that

$$(\text{Number of columns} - 1)/N = \text{a whole number}$$

where $N=2$ for one-plane symmetry and $N=4$ for two-plane symmetry.

As mentioned before, each line segment has a constant circulation Γ_i. These circulations can be considered the unknowns. However, it is more convenient to replace them with the circulations around constant-strength loop vortices which enclose each element, G_i. Then the circulations around the individual segments along the sides of the elements can be found from algebraic combinations of the circulations around the loops. Using closed loops of constant-circulation vortex filaments, we satisfy the spatial conservation of circulation. Moreover, the influence coefficient matrix becomes strongly diagonal. For the source panels we take the strength of each source element σ_i so that the velocity field it generates cancels part of the normal component of the freestream velocity on the same element. Since the normal velocity induced by a source element of unit strength on itself is equal to 2π, then

$$(2\pi)\sigma_j = \beta(FS_j)$$

or

$$\sigma_j = \frac{\beta}{2\pi} FS_j \tag{4}$$

where β is the fraction of the normal component of the freestream velocity to be cancelled and FS_j is the normal component of the freestream velocity on the jth element. The factor β is constant for all elements on the same body. It is arbitrary (ranges between $0 \rightarrow 1.0$). The system of equations that represent the no-penetration boundary conditions for all

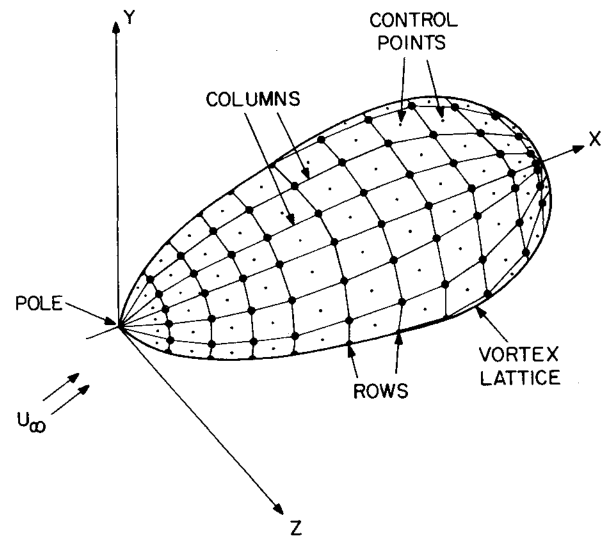

Fig. 1 Division of the surface into elements.

elements on the body can be written in index notation as follows:

$$\sum_{j=1}^{NT} (AV_{ij}G_j + AS_{ij}\sigma_j) = -FS_i \quad \text{for } i=1,2,\ldots,NT$$

Since all source strengths are predetermined, the above equation becomes

$$\sum_{j=1}^{NT} AV_{ij}G_j = -FS_i - \sum_{j=1}^{NT} AS_{ij}\sigma_j \tag{5}$$

where

NT = the total number of elements on the body surface.
AV_{ij} = the normal component of velocity at the control point of the ith receiving element due to a unit loop circulation around the jth sending element.
AS_{ij} = the normal component of velocity at the control point of the ith receiving element due to a unit source strength at the jth sending element.
FS_i = the normal component of the freestream velocity at the control point of the ith element.

The choice of the control points, where the no-penetration boundary condition is satisfied, is crucial due to the singular nature of the vortex lines. The best results were obtained when the control points were located so that the self-induced velocity is a minimum. These points are located by a search technique capable of treating spatially skewed elements. This is accomplished by projecting the element on a plane defined by the centroid of the corners of the element and a unit normal defined by the cross product of the diagonal vectors of the actual element. Then, the search is performed on the projected element.

Since a large number of elements is required to represent a general body, the high-speed memory of the computing machine may not be able to handle the large influence matrix involved, and consequently, some equation solvers cannot be used conveniently. In this case, the entire coefficient matrix can be stored on a disk where it can be retrieved and transferred to the high-speed memory but with limited portions at a time. We found that the modified Gauss-Seidel iteration technique is well-suited to the general problem because it deals with one equation at a time or, equivalently, one row of the big matrix at a time. The choice of loop circulations for the unknowns and the points of minimum self-induced velocity for control points apparently renders the influence matrix positive definite, and we did not experience any trouble with convergence.

For symmetric flows the set of equations can be reduced. In some cases, other equation solvers, such as Gauss elimination and matrix inversion, are used to solve the reduced set of equations.

Having solved the equations for the unknown circulations, we are in a position to calculate the pressure at each control point on the surface of the body. The pressure coefficient is calculated by using Bernoulli's equation:

$$C_p = 1 - V_T^2/U_\infty^2 \tag{6}$$

where C_p is the local pressure coefficient, V_T is the total tangential velocity of the flow at the control point on the surface of the body, and U_∞ is the freestream velocity taken to be of unit magnitude in the dimensionless problem.

The total tangential velocity at each control point is evaluated as the sum of three parts. First, the part due to the influence on the control point of all the elements in the lattice (vortices and sources) is evaluated. Second, the contribution of the freestream velocity is obtained. Third, the velocity jump across the vortex sheet is determined.

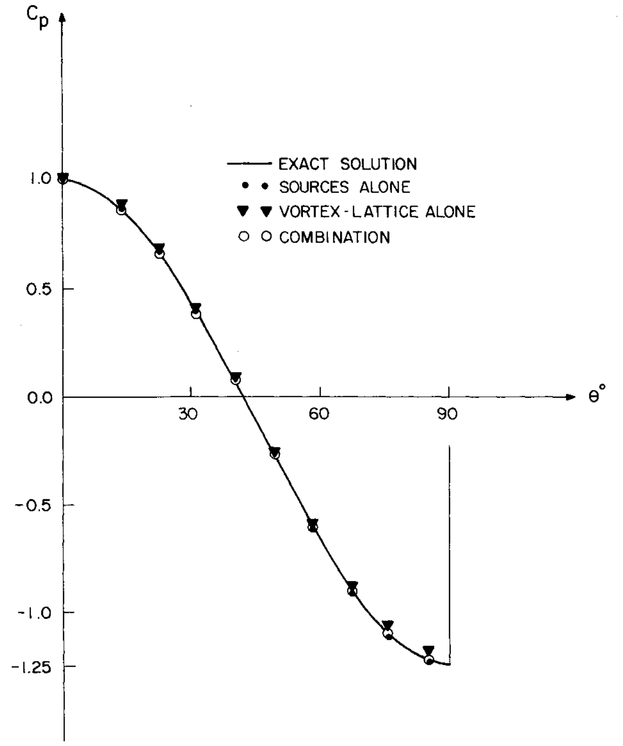

Fig. 2 Pressure coefficient as a function of position on a sphere.

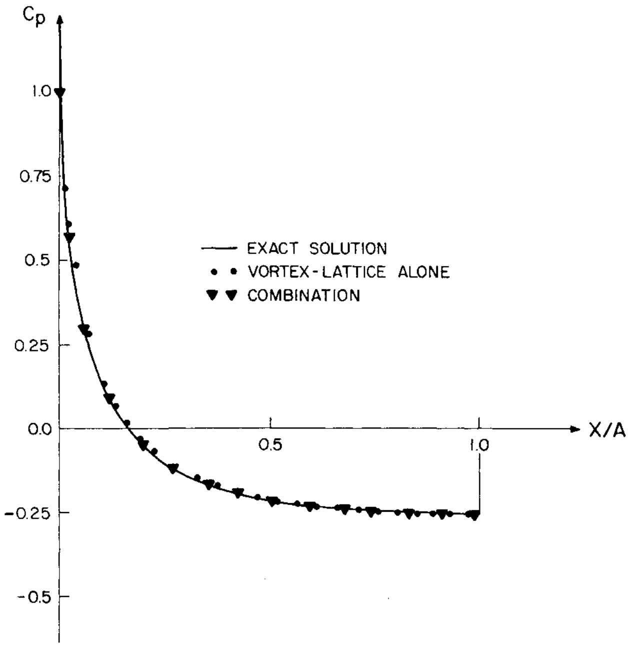

Fig. 3 Pressure coefficient as a function of position on an ellipsoid of revolution.

The program can calculate the velocities and pressures at points other than those on the body with a slight modification in the pressure routine.

Numerical Examples

The present method was tested and compared with analytical solutions and/or experimental data. In the following, each test case is presented and discussed.

Figure 2 shows a comparison of the present solution with the exact solution, the sources alone, and the vortex lattice alone, for flow around a sphere. The results shown were calculated using the axisymmetric option of the program.

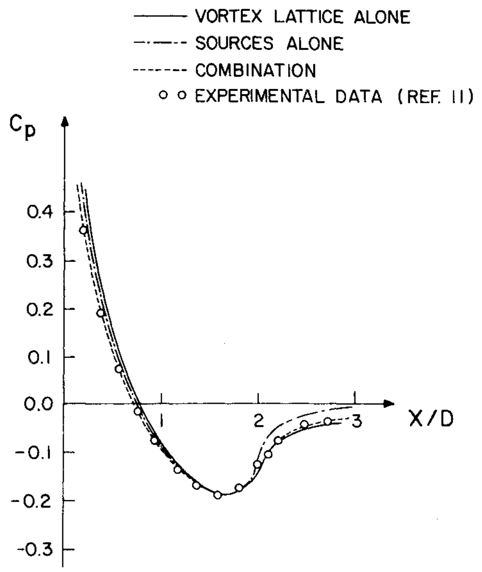

Fig. 4 Pressure coefficient as a function of position on an ogive-cylinder.

Twenty equidistant ring elements were used in all cases. It is noted that the combination yields more accurate results than either the vortex lattice alone or the sources alone. The maximum error in the pressure coefficient calculated by the combination is less than 2%. Due to symmetry, only half the curve is shown. Using the general program and an angle of attack of 90 deg, the same results were reproduced at corresponding points. Figure 3 shows a comparison of the present solution with the exact solution for flow around an ellipsoid of revolution. The fineness ratio is three, which represents the transition from a blunt to a streamlined body. For the combination of sources and vortex lattice, only thirty ring elements were used; while for the vortex lattice alone, fifty-four equidistant ring elements were used. The results are in good agreement with the exact solution, even in the high-curvature region where the vortex-lattice technique is not very accurate. The spacing between ring elements is smaller near the nose than it is in the midsection. Figure 4 shows a comparison of the results obtained by the combination, the vortex lattice alone, the sources alone, and the experimental data (taken from Faulkner et al.[11]) for axisymmetric flow past an ogive-cylinder. The slenderness ratio is 2.0. In all three cases, 400 elements were used. The present results compare better with the experimental data than the individual surface-singularity methods for the same number of elements. The predicted pressures are insensitive to the distance covered along the cylinders. In this case and the next one, we found that the location of the control points is critical, especially in the nose region, where the elements are somewhat elongated. Using the point of minimum self-induced velocity speeded the convergence in the Gauss-Seidel iteration.

Figures 5a, 5b and 5c show a comparison of the pressures predicted by the present method with those predicted by the method of Ref. 1 and available experimental data for a cone-cylinder. Two cases were considered: pressures along the cone and along the cylinder. At zero angle of attack (Fig. 5a), all three pressures agree closely along the cone. But along the cylinder, the combination works much better than the method of Ref. 1. At 20 deg angle of attack, along the high-pressure side of the cone (Fig. 5b), all are in good agreement again. But along the cylinder, the present results agree much better with the experimental data than those of Ref. 1. Along the low-pressure side of the body (Fig. 5c), the results of Ref. 1 agree better with the experimental data near the nose but are worse near the shoulder. Along the cylinder, the two results are

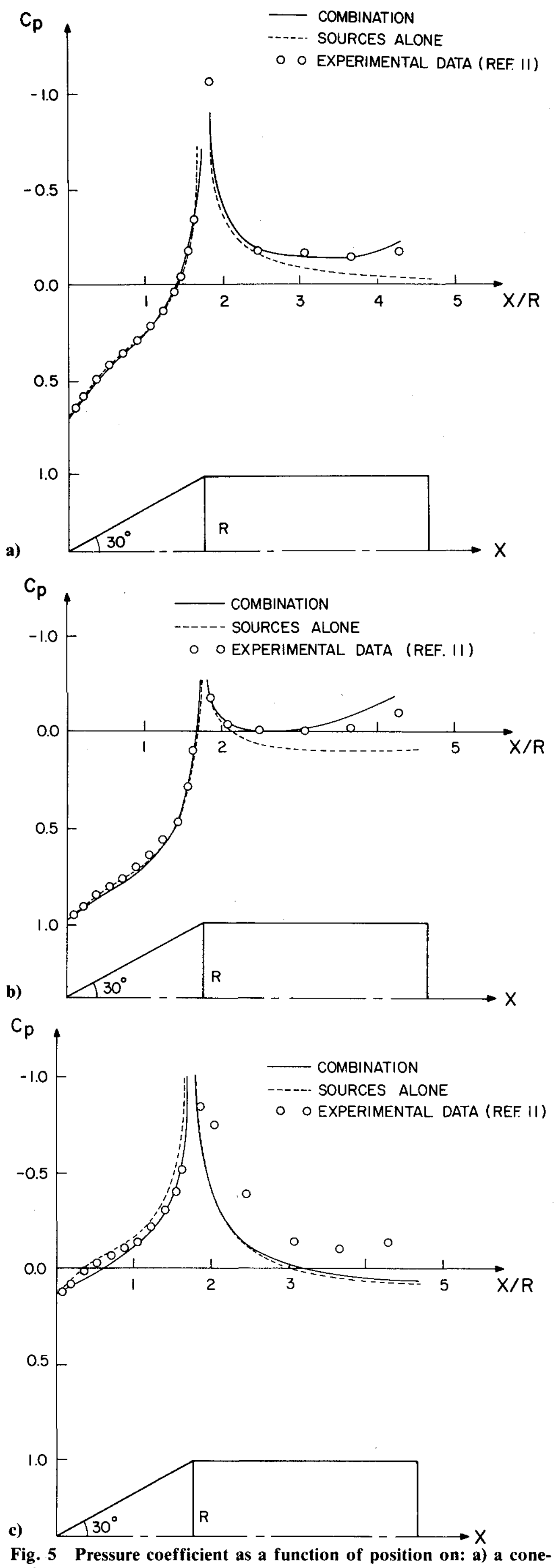

Fig. 5 Pressure coefficient as a function of position on: a) a cone-cylinder, angle of attack = 0 deg; b) windward side of a cone-cylinder, angle of attack = 20 deg; and c) leeward side of a cone-cylinder, angle of attack = 20 deg.

nearly identical, and none agrees well with the experimental data. This is the separated region on the body. It is worthwhile to note that no attempt was made to add an afterbody which would simulate the wake. Thus, in the after region, the predicted pressures could be strongly influenced by the sharp corner.

III. Concluding Remarks

The present results indicate that a vortex lattice alone can be used to model the flow past an arbitrary three-dimensional body at least as well as, and in many cases better than, a source distribution alone. The choice of control point appears to be crucial, and the present results indicate that the point of minimum self-induced velocity yields the best results. Moreover, the present results indicate that for blunt bodies it is advantageous to combine the vortex lattice with a surface distribution of sources. This can be done with practically no additional effort by specifying the strength of the source distribution. It appears that $0.3 \leq \beta \leq 0.7$ [see Eq. (4)] yields the best results.

Experience has shown that for slender bodies the coefficient β should be less than 0.1. Otherwise, it appears that the sources in the region where the surface is nearly parallel to the freestream strongly bias the right-hand side of Eq. (5). The combination with $\beta < 0.1$ did not have an appreciable advantage over the vortex lattice alone. However, the latter takes less computation time.

Acknowledgments

This reserach was supported by the Army Research Office under Grant No. DAAG-76-G-0064. The many helpful suggestions by E. Carson Yates are appreciated.

References

[1] Hess, J.L. and Smith, A.M.O., "Calculation of Potential Flow about Arbitrary Bodies," *Progress in Aeronautical Sciences*, Vol. 8, Pergamon Press, New York, 1966.

[2] Hess, J.L. and Martin, R.P., Jr., "Improved Solution for Potential Flow about Arbitrary Axisymmetric Bodies by the Use of a Higher-Order Surface-Source Method," NACA-CR-134694, 1974.

[3] Maskew, B., "Calculation of the 3-D Potential Flow around Lifting Non-Planar Wings and Wing-Bodies Using a Surface Distribution of Quadrilateral Vortex-Rings," Dept. of Transport Technology, Loughborough University of Technology, TT 7009, Sept. 1970.

[4] Atta, E. and Nayfeh, A.H., "Nonlinear Aerodynamics of Wing-Body Combinations," AIAA Paper 78-1206, Seattle, Wash., July 1978.

[5] Geissler, W., "Calculation of Potential Flow about Axially Symmetric Fuselages, Annular Profiles and Engine Inlets," *Z. Flugwisenschaften*, Vol. 20, Dec. 1972, pp. 457-462.

[6] Bristow, D.R., "A New Surface Singularity Method for Multi-Element Airfoil Analysis and Design," AIAA Paper 76-20, Washington, D.C., Jan. 1976.

[7] Uchiyama, N., Mikkilineni, R.P. and Wu, J.M., "The Analysis of Wing-Body Combinations at Moderate Angles of Attack," AIAA Paper 78-62, Huntsville, Ala., Jan. 1978.

[8] Nikolitsch, D., "Normal Force and Pitching Moment Coefficients of Bodies and Wing-Body Combinations at Very High Angles of Attack," AIAA Paper 78-63, Huntsville, Ala., Jan. 1978.

[9] Morinò, L., "A Finite Element Formulation for Subsonic Flows around Complex Configuration," NACA-CR-138142, 1973.

[10] David, A.L., and Geppson, R.W., "Solving 3-D Potential Flow Problems by Means of an Inverse Formulation and Finite Differences," NTIS CSCL 20/4, N73-26302 (Microfiche), 1973.

[11] Faulkner, S., Hess, J.L. and Giesing, J.P., "Comparison of Experimental Pressure Distributions with Those Calculated by the Douglas Neuman Program," Douglas Aircraft Company, Rept. LB31831, Dec. 1964.